Answer Key to PRACTICE QUESTIONS SET – 4 (2024-25)

MATHEMATICS

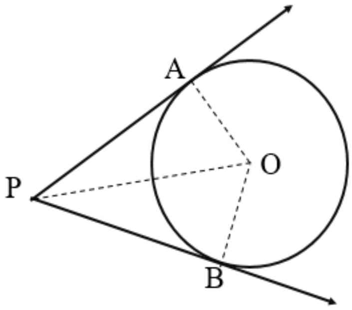
CLASS – X

Section A

Answer ALL the following Questions. Each question carries 2 marks.

1. **Prove that the lengths of tangents drawn from an external point to a circle are equal.**

Answer:



Given: In circle, O is the centre. P is an external point and PA and PB are tangents drawn.

To Prove: $PA = PB$

Construction: Join OA, OB and OP

Proof: Since PA and PB are the tangents and OA and OB are the radii of a circle, therefore $OA \perp PA$ and $OB \perp PB$

$\Rightarrow \angle OAP = \angle OBP = 90°$

In ΔOAP and ΔOBP,

$OA = OB$ (radii) , $OP = OP$ (common) , $\angle OAP = \angle OBP = 90°$

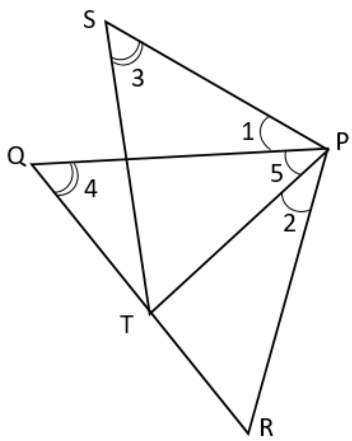
$\therefore \Delta OAP \cong \Delta OBP$(by RHS congruency rule)

$PA = PB$ (by cpct)

proved

2. **In the figure, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Show that $PT \times QR = PR \times ST$**

Answer: $\angle 1 = \angle 2$ (Given)



$\Rightarrow \angle 1 + \angle 5 = \angle 2 + \angle 5$

$\Rightarrow \angle SPT = \angle QPR$

And $\angle 3 = \angle 4$

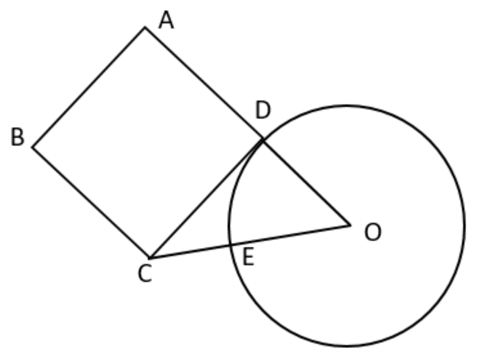
$\Rightarrow \Delta PST \sim \Delta PQR$

$\Rightarrow \frac{ST}{QR} = \frac{PT}{PR}$

(Corresponding sides of similar triangles are proportional)

$\Rightarrow PT \times QR = PR \times ST$

3. **ABCD is a square. CD is a tangent to the circle with centre O as shown in the figure. If OD = CE, what is the ratio of the area of the circle and the area of the square? Give reasons for your answer.**



Answer:

Here, OE = OD (radii of the circle)

CE = OD

OC = CE + OE = OD + OD = 2OD

Also CD is tangent to the circle. Thus OD is perpendicular to CD.

(radius through the point of contact of tangent)

$\Rightarrow \angle ODC = 90°$

In right $\Delta ODC, OC^2 = OD^2 + CD^2$

$\Rightarrow (2OD)^2 = OD^2 + CD^2$

$\Rightarrow 3OD^2 = CD^2$

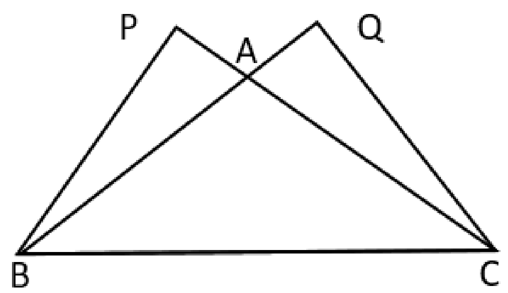
$\Rightarrow CD = \sqrt{3}\,OD$

Area of the square $= CD^2 = 3(OD)^2$

Area of the circle $= \pi r^2 = \pi(OD)^2$

Area of the circle : Area of the square = $\pi(OD)^2 : 3(OD)^2 = \pi : 3$

4. **In ΔABC, $\angle A$ is obtuse, $PB \perp AC$ and $QC \perp AB$. Prove that $AB \times AQ = AC \times AP$**



Answer:

In ΔAPB and ΔACQ

$\angle BPA = \angle CQA = 90°$

$\angle BAP = \angle CAQ$ (Vertically opposite angles)

$\therefore$ $\Delta APB \cong \Delta AQC$ (by AA similarity)

$\frac{AP}{AQ} = \frac{AB}{AC}$

$\Rightarrow$ $AB \times AQ = AC \times AP$

Proved.

5. **If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$.**

Answer:

BQ = BP , CP = CR and AQ = AR

Now, 2AQ = AQ + AR

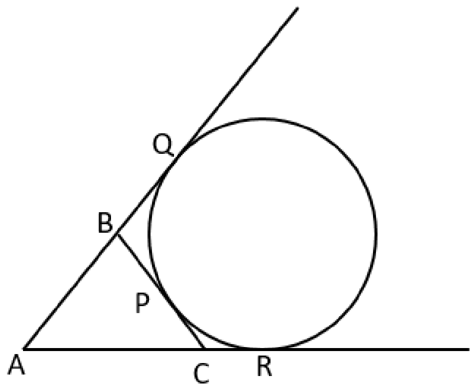
= (AB + BQ) + (AC + CR)

= AB + BP + AC + CP

= (BP + CP) + AC + AB

= BC + CA + AB

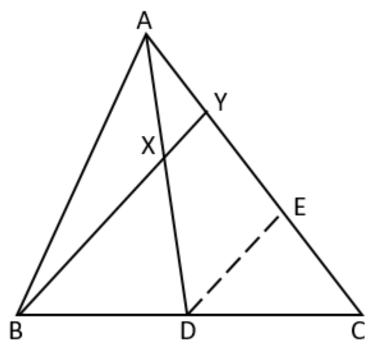
Therefore, $AQ = \frac{1}{2}(BC + CA + AB)$.



Section B

Answer ALL the following Questions. Each question carries 3 marks.

6. **In a ΔABC, AD is a median. X is a point on AD such that $AX: AD = 2: 3$. Ray BX intersects AC in Y. Prove that BX = 4XY.**



Answer:

Given: ΔABC, AD is a median. $AX : AD = 2 : 3$

To Prove: BX = 4XY

Construction: Construct DE || BY

If $XY = p$

By Thale's Theorem, $\frac{AX}{AD} = \frac{XY}{DE} \Rightarrow \frac{2}{5} = \frac{p}{DE} \Rightarrow DE = \frac{5}{2}p$

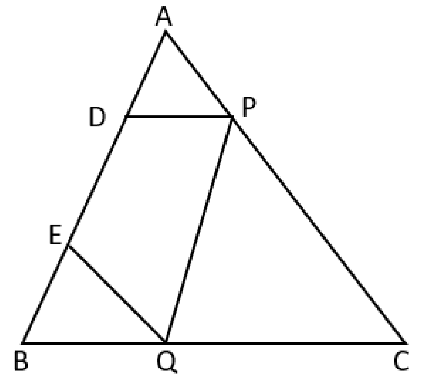
Now in ΔBYC, DE || BY$\Rightarrow \frac{DC}{BC} = \frac{DE}{BY}$

$\Rightarrow \frac{1}{2} = \frac{\frac{5}{2}p}{BY} = BY = 5p$

Now, $BX = BY - XY = 5p - p = 4p$

As $\frac{BX}{XY} = \frac{4p}{p} \Rightarrow BX = 4XY$ Proved

7. **In the figure, there are two points D and E on side AB of ΔABC such that AD = BE. If DP || BC and EQ || AC, prove that PQ || AB.**



Answer:

Given that In $\Delta ABC, DP||BC \therefore \frac{AD}{BD} = \frac{AP}{PC}$ (by Thale's Theorem) $\Rightarrow \frac{AD}{AB} = \frac{AP}{AC}$ ------(i)

Similarly, In $\Delta ABC, EQ||AC \therefore \frac{BQ}{BC} = \frac{BE}{BA} \Rightarrow \frac{BQ}{BC} = \frac{AD}{AB}$ (as $AD = BE$) ------(ii)

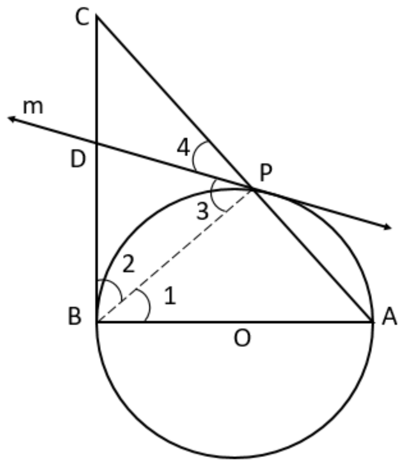
From (i) & (ii), $\frac{BQ}{BC} = \frac{AP}{AC} \Rightarrow \frac{BQ}{QC} = \frac{AP}{PC}$

By converse of Thale's theorem, PQ || AB

Proved.

8. **In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC.**

Answer:



Given: O is the centre of the circle drawn with AB as a diameter. A tangent m to the circle at P intersects BC at D.

To Prove: BD = DC

Proof:

As AB is a diameter of the circle, APB is a semi-circle.

Thus, ΔAPB is a right triangle at P.

$\therefore \angle 1 + \angle A = 90°$ ------(i)

In ΔABC, $\angle ABC = 90° \Rightarrow OB \perp BC \Rightarrow$ BC is tangent to the circle at B.

Again, , $\angle ABC = 90° \Rightarrow \therefore \angle 1 + \angle 2 = 90°$ --------(ii)

From (i) and (ii), $\angle A = \angle 2$-------(iii)

As DP and DB are tangents drawn from D to the circle, $DP = DB \Rightarrow \angle 2 = \angle 3$-----(iv)

From (iii) & (iv), $\angle A = \angle 3$

Now $\angle APB = 90° \Rightarrow \angle BPC = 90° \Rightarrow \angle 3 + \angle 4 = 90°$ --------(v)

In right $\Delta ABC, \angle A + \angle C = 90°$------(vi)

From (v) & (vi), $\angle 3 + \angle 4 = \angle A + \angle C$

$\Rightarrow \angle C = \angle 4$

$\Rightarrow DP = DC$

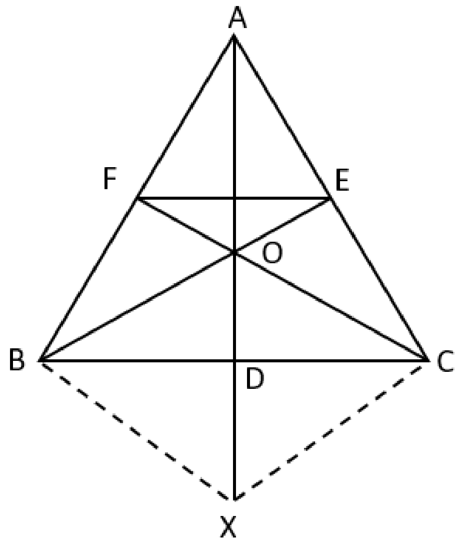
$\Rightarrow DB = DC$

Hence m bisects BC.

9. **The side BC of a ΔABC is bisected at D; O is any point in AD. BO and CO produced meet AC and AB in E and F respectively and AD is produced to X so that D is the midpoint of OX. Prove that**

a) $\mathbf{AO : AX = AF : AB}$

b) FE || BC



Answer:

Construction: Join BX and CX.

Proof: BD = CD and OD = DX

⇒ BC and OX bisect each other.

⇒ OBXC is a parallelogram.

⇒ BX || OC and CX || OB

⇒ BX || CF and CX || BE

⇒ BX || OF and CX || OE

In $\Delta\mathbf{ABX}$, BX || OF, by BPT, $\frac{AO}{AX} = \frac{AF}{AB}$ -----(i)

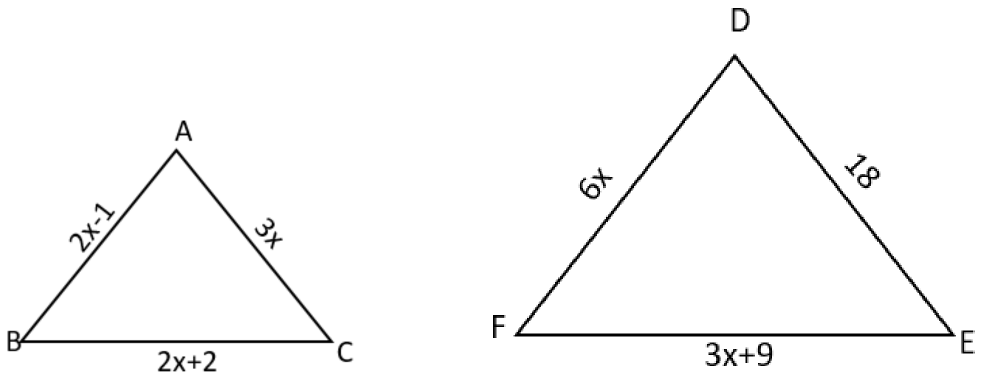
In $\Delta\mathbf{ACX}$, CX || OE, by BPT, $\frac{AO}{AX} = \frac{AE}{AC}$ -----(ii)

From (i) & (ii), $\frac{AF}{AB} = \frac{AE}{AC}$

Therefore, by converse of BPT, FE || BC.

10. **In the given figure, ΔABC~ΔDEF and their sides are of lengths (in cm) as marked along their sides, find the lengths of the sides of each triangle.**

Answer:



As ΔABC~ΔDEF, therefore $\frac{AB}{DE} = \frac{AC}{DF}$

$\Rightarrow \frac{2x-1}{18} = \frac{3x}{6x} = \frac{1}{2}$

$\Rightarrow \frac{2x-1}{18} = \frac{1}{2}$

$\Rightarrow 4x - 2 = 18$

$\Rightarrow x = 5$

$\therefore AB = (2 \times 5 - 1)cm = 9\ cm, BC = (2 \times 5 + 2)cm = 12cm,$

$AC = (3 \times 5)cm = 15cm, EF = (3 \times 5 + 9)cm = 24cm$ and $DF = (6 \times 5)cm = 30\ cm$

Section C

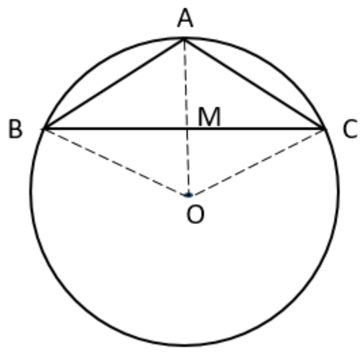
Answer ALL the following Questions. Each question carries 5 marks.

11. **If an isosceles ΔABC in which $AB = AC = 6$ cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.**

Answer:

Given: In a circle, ΔABC is inscribed.

To find: Area of ΔABC



Construction: Join OB, OC and OA

In ΔABO and ΔACO,

$AB = AC$(given)

$BO = CO$ (radii of same circle)

AO is common.

Therefore, $\Delta ABO \cong \Delta ACO$ (by SSS congruency)

$\Rightarrow \angle BAO = \angle CAO$ (by cpct)

In ΔAMB and ΔACM,

$AB = AC$ (given), $\angle BAO = \angle CAO$ (proved above), AM is common

Therefore, $\Delta AMB \cong \Delta ACM$ (by SAS congruency)

$\Rightarrow \angle AMB = \angle AMC$ (cpct)

And $BM = MC$(cpct)

But $\angle AMB + \angle AMC = 180° \Rightarrow \angle AMB = \angle AMC = 90°$

Thus, OA is perpendicular bisector of BC.

Let $AM = x$, then $OM = 9 - x$

In right ΔAMC, $MC^2 = 6^2 - x^2$------(i)

In right ΔOMC, $MC^2 = 9^2 - (9 - x)^2$-------(ii)

From (i) & (ii), $6^2 - x^2 = 9^2 - (9 - x)^2 \Rightarrow x = 2$

$\therefore AM = x = 2$

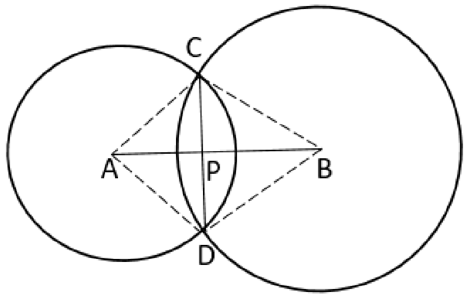
In right $\Delta ABM, BM^2 = 6^2 - 2^2 = 32 \Rightarrow BM = 4\sqrt{2} \Rightarrow BC = 2BM = 8\sqrt{2}$ cm

Area of $\Delta ABC = \frac{1}{2} \times BC \times AM = \frac{1}{2} \times 8\sqrt{2} \times 2 = 8\sqrt{2}\ cm^2$

12. **Two circles with centres A and B of radii 6 cm and 8 cm respectively intersect at two points C and D such that AC and BC are tangents to the two circles. Find the length of the common chord CD.**

Answer:

Tangent to a circle at a point is perpendicular to radius at point of contact.



$\therefore \angle ACB = 90^\circ$

In ΔACB, we have

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = 6^2 + 8^2 = 100 \Rightarrow AB = 10$ cm

Since the line joining the centres of two intersecting circles is perpendicular bisector of their common chord, so

$AP \perp CD$ and $CP = PD$

Let $AP = x$ cm, then $BP = (10 - x)$cm

Let $CP = DP = y$ cm

In ΔAPC and ΔBPC,

$AC^2 = AP^2 + PC^2, BC^2 = PB^2 + PC^2$

$\Rightarrow 6^2 = x^2 + y^2$ and $8^2 = (10 - x)^2 + y^2$

$\Rightarrow 8^2 - 6^2 = (10 - x)^2 + y^2 - x^2 - y^2$

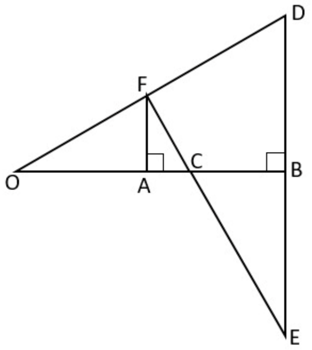
$\Rightarrow 28 = 100 - 20x$

$\Rightarrow x = 3.6$ cm

As, $6^2 = x^2 + y^2 \Rightarrow y = \sqrt{36 - (3.6^2)} = 4.8$ cm

Length of the common chord $CD = 2\ CP = 2y = 9.6$ cm

13. **OB is perpendicular bisector of the line segment DE, FA ⊥ OB and FE intersects OB at the point C. Prove that (a) $\frac{OA}{OB} = \frac{AC}{BC}$ (b) $\frac{1}{OA} + \frac{1}{OB} = \frac{2}{OC}$**



Answer:

In ΔAOF and ΔBOD,

$\angle AOF = \angle BOD$ (common) and $\angle OAF = \angle OBD = 90°$

By AA similarity, $\Delta AOF \sim \Delta BOD \Rightarrow \frac{OA}{OB} = \frac{FA}{DB}$ ----(I)

In ΔFAC and ΔEBC,

$\angle FAC = \angle EBC$ (common) and $\angle ACF = \angle BCE = 90°$

By AA similarity, $\Delta FAC \sim \Delta EBC \Rightarrow \frac{FA}{EB} = \frac{AC}{BC} \Rightarrow \frac{FA}{DB} = \frac{AC}{BC}$ -----(II)

From (I) & (II), $\frac{OA}{OB} = \frac{AC}{BC}$, proved .

$\Rightarrow \frac{OA}{OB} = \frac{OC-OA}{OB-OC} \Rightarrow OA(OB - OC) = OB(OC - OA)$

$\Rightarrow 2OA.OB = OB.OC + OC.OA$

$\Rightarrow (OB + OA)OC = 2OA.OB$

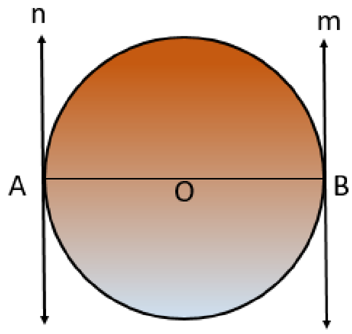
$\Rightarrow \frac{1}{OA} + \frac{1}{OB} = \frac{2}{OC}$, proved.

Section – D : Case Study

14. **A circle can have at most two parallel tangents, one at a point on it and the other at a point diametrically opposite to it. Here AB is diameter of a circle and tangent n and**

tangent m drawn at the points A and B respectively are parallel to each other.

Answer the questions based on above.



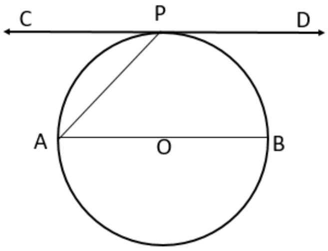
a) What is the distance between two parallel tangents of a circle if radius 6 cm?

b) What is the maximum number of tangents parallel to a secant a circle?

c) Two parallel tangents touch the circle A and B. Find the distance between parallel tangents if the area of circle is $25\pi\ \text{cm}^2$.

OR

CD is tangent to circle at P. If $\angle PAB = 25°$, determine $\angle CPA$ given O is the centre of the circle.



Answer:

a) The distance between two parallel tangents of a circle of radius 6 cm

= Diameter of a circle

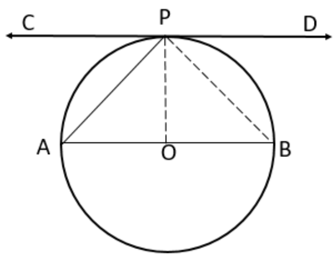
= 12 cm

b) Maximum number of parallel to a secant can be two.

c) Area of the circle $= 25\pi\ \text{cm}^2 \Rightarrow \pi r^2 = 25\pi \Rightarrow r = 5\ \text{cm}$

Distance between the parallel tangents = 2 x 5 = 10 cm

OR



In $\Delta APO, \angle PAO = 25°$

As $OP = OA, \angle PAO = \angle APO = 25°$

$\angle CPA = \angle CPO - \angle APO = 90° - 25° = 65°$

15. **ASSERTION REASON BASED QUESTIONS**

A statement of assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Assertion(A): In ΔABC, D and E are points on sides AB and AC respectively such that BD = CE. If $\angle B = \angle C$, then DE is not parallel to BC.

Reason(R): If a line divides any two sides of a triangle in the same ratio, then line must be parallel to the third side.

Answer: (d)