Answer Key to PRACTICE QUESTIONS SET – 1 (2024-25)

MATHEMATICS

CLASS – X

Section A

Answer ALL the following Questions. Each question carries 2 marks.

1. **If the H.C.F of 408 and 1032 is expressible in the form of $1032 \times 2 + 408 \times p$, find the value of p.**

Answer:

$1032 \times 2 + 408 \times p = 24$

$\Rightarrow 408p = 24 - 2064$

$\Rightarrow p = -5$

2. **If one zero of the polynomial $f(x) = 3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.**

Answer:

Let α and 7α be the zeroes of polynomial.

Then $\alpha + 7\alpha = \frac{8}{3}$

$\Rightarrow 8\alpha = \frac{8}{3} \Rightarrow \alpha = \frac{1}{3}$

Therefore, the two zeroes are $\frac{1}{3}$ and $\frac{7}{3}$

Now, $\frac{7}{3} \times \frac{1}{3} = \frac{2k+1}{3}$

$\Rightarrow \frac{7}{3} = 2k + 1 \Rightarrow k = \frac{2}{3}$

3. **Show that 12^n cannot end with digit 0 or 5 for any natural number n.**

Answer:

$12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$

For a number to end with digit 0 or 5, its prime factorization must contain the factor 5. But we see that, the prime factorization of 12^n does not contain the factor 5.

So 12^n cannot end with digit 0 or 5.

4. **Explain why $11 \times 17 \times 5 \times 3 \times 2 \times 1 + 5$ is a composite number.**

Answer:

$11 \times 17 \times 5 \times 3 \times 2 \times 1 + 5$

$= 5 \times (11 \times 17 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1122 + 1) = 5 \times 1123$

The given expression can be expressed as a product of two numbers other than 1 and itself.

Hence, $11 \times 17 \times 5 \times 3 \times 2 \times 1 + 5$ is a composite number.

5. **If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + 3x - 2$, evaluate $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$**

Answer:

Here, $\alpha + \beta = -3$, $\alpha\beta = -2$

$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$

$= \frac{[(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)]}{\alpha\beta}$

$= \frac{[(-3)^3-3(-2)(-3)]}{-2} = \frac{-45}{-2} = \frac{45}{2}$

6. **Prove that $\frac{(\sqrt{5}-2)}{3}$ is an irrational number, given that $\sqrt{5}$ is an irrational number.**

Answer:

Let us assume $\frac{(\sqrt{5}-2)}{3} = \frac{a}{b}$ be a rational number such that a and b are co-prime integers and $b \neq 0$

Therefore, $\sqrt{5} = \frac{3a}{b} + 2 \Rightarrow \sqrt{5} = \left(\frac{3a+2b}{b}\right)$

Here, as a and b are integers, R.H.S is rational but $\sqrt{5}$ is irrational.

This is a contradiction. Thus, our assumption is incorrect.

Hence, $\frac{(\sqrt{5}-2)}{3}$ is an irrational number.

Section B

Answer ALL the following Questions. Each question carries 3 marks.

7. **Find the smallest pair of 4-digit positive numbers such that the difference between them is 303 and their H.C.F is 101.**

Answer:

Let x and y be the required smallest pair and $x < y$.

Given that $y - x = 303$

The smallest 4-digit number is 1000.

101)1000(9

- 909

91 (Remainder)

Therefore, $x = 1000 + (101 - 91) = 1010$

Thus, $y - 1010 = 303 \Rightarrow y = 1313$

Hence, required pair of smallest 4-digit numbers are 1010 and 1313.

8. **If $(x - k)$ is the H.C.F of $(2x^2 - kx - 9)$ and $(x^2 + x - 12)$, find the value of k.**

Answer:

Let, $p(x) = (x^2 + x - 12) = (x + 4)(x - 3)$

Thus k can be either -4 or 3 -------(I)

As $(x - k)$ is the H.C.F of $q(x) = (2x^2 - kx - 9)$, $x = k$ will satisfy $q(x)$ i.e, $q(k) = 0$.

$Q(k) = (2k^2 - k^2 - 9)$

$\Rightarrow k^2 - 9 = 0$

$\Rightarrow k = \pm 3$ ----------(II)

Therefore, from (I) & (II), $k = 3$

9. **Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $\left(-\frac{3}{2}\right)$, respectively. Also find its zeroes.**

Answer:

The quadratic polynomial is $x^2 - \sqrt{2}x - \frac{3}{2}$

Now, $x^2 - \sqrt{2}x - \frac{3}{2} = \frac{1}{2}[2x^2 - 2\sqrt{2}x - 3]$

$= \frac{1}{2}[2x^2 + \sqrt{2}x - 3\sqrt{2}x - 3]$

$= \frac{1}{2}[\sqrt{2}x(\sqrt{2}x + 1) - 3(\sqrt{2}x + 1)$

$= \frac{1}{2}\left[\left(\sqrt{2}x - 3\right)\left(\sqrt{2}x + 1\right)\right]$

Therefore, the zeroes are $-\frac{1}{\sqrt{2}}$ and $\frac{3}{\sqrt{2}}$

10. **Prove that $\sqrt{3}$ is an irrational number.**

Answer:

Let $\sqrt{3}$ be a rational number such that $\sqrt{3} = \frac{a}{b}$, where a and b are co-primes and $b \neq 0$.

Now, $\left(\sqrt{3}\right)^2 = \left(\frac{a}{b}\right)^2 \Rightarrow a^2 = 3b^2$ --------(I)

$\Rightarrow$ 3 divides a^2 $\therefore$ 3 divides a.

Let $a = 3c$,where c is any integer.

Substituting in (I), $(3c)^2 = 3b^2 \Rightarrow 3c^2 = b^2 \Rightarrow$ 3 divides b^2 $\therefore$ 3 divides b.------(II)

From (I) & (II), 3 is a common factor of a and b which contradicts a and b are co-primes.

So our assumption is incorrect.

Hence, $\sqrt{3}$ is an irrational number.

11. **The number of students learning German, Arabic and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of students are seated and all of them are of the same subject.**

Answer:

Number of students in each room = H.C.F of 48, 80 and 144

$48 = 2^4 \times 3^1, \quad 80 = 2^4 \times 5^1, \quad 144 = 2^4 \times 3^2$

$\therefore$ H.C.F of 48, 80 and 144 is $2^4 = 16$

Therefore, 16 students can be seated in each room

Number of rooms required $= \frac{48+80+144}{16} = 17$

Section C

Answer ALL the following Questions. Each question carries 5 marks.

12. **Find the zeroes of the quadratic polynomial $p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients. Hence, form a quadratic polynomial whose sum and product of zeroes are same as $p(y)$.**

Answer:

$p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$= \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}(7y + 1)(3y - 2)$

For zeroes, $p(y) = 0 \Rightarrow y = -\frac{1}{7}, \frac{2}{3}$

Now, $a = 7, b = -\frac{11}{3}$, $c = -\frac{2}{3}$

Sum of zeroes $= -\frac{1}{7} + \frac{2}{3} = \frac{11}{21}$

Also, Sum of zeroes $= -\frac{b}{a} = -\frac{-\frac{11}{3}}{7} = \frac{11}{21}$ Verified

Product of zeroes $= -\frac{1}{7} \times \frac{2}{3} = \frac{-2}{21}$

Also, product of zeroes $= \frac{c}{a} = \frac{\frac{2}{3}}{7} = \frac{2}{21}$ Verified

The required polynomial is $k\left(x^2 - \frac{11}{21}x + \frac{2}{21}\right)$, where k is any non - zero real number.

13. **If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx + 8x - 9$ is negative of the other, find the zeroes of $kx^2 + 3kx + 2$.**

Answer:

$f(x) = 4x^2 - 8kx + 8x - 9 = 4x^2 - 8(k-1)x - 9$

Let α and $-\alpha$ be the two zeroes.

Now, $\alpha + (-\alpha) = \frac{[-8(k-1)]}{4} \Rightarrow k = 1$

$p(x) = kx^2 + 3kx + 2 = x^2 + 3x + 2 = (x+2)(x+1)$

Therefore, the zeroes of $p(x)$ are (-2) and (-1).

14. **Find the H.C.F and L.C.M of 180 and 288 by prime factorisation method. Also, verify that HCF x LCM = Product of two given numbers.**

Answer:

$180 = 2^2 \times 3^2 \times 5$

$288 = 2^5 \times 3^2$

$H.C.F(180, 288) = 2^2 \times 3^2 = 36$

$L.C.M(180, 288) = 2^5 \times 3^2 \times 5 = 1440$

$H.C.F(180, 288) \times L.C.M(180, 288) = 36 \times 1440 = 51{,}840$

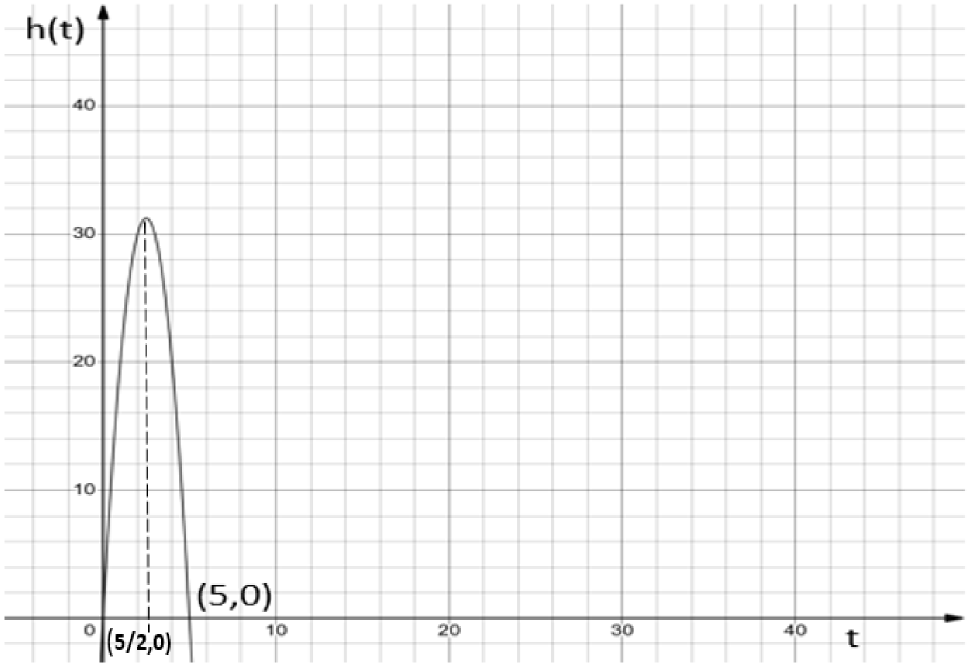
Product of two given numbers = 180 x 288 = 51,840

Therefore, HCF x LCM = Product of two given numbers

Verified.

Section – D : Case Study

15. **A ball is thrown in the air so that t seconds after it is thrown, its height h metre above its starting point is given by the polynomial $h(t) = 25t - 5t^2$.**



Observe the graph of the polynomial and answer the following questions:

(i) Write zeroes of the given polynomial.

(ii) Find the maximum height achieved by ball.

(iii) (a) After throwing upward, how much time did the ball take to reach to the height of 30m?

OR

(b) Find the two different values of t when the height of the ball was 20m?

Answer:

(i) $h(t) = 25t - 5t^2$

Substituting $h(t) = 0$, we get $25t - 5t^2 = 0 \Rightarrow 5t(5 - t) = 0 \Rightarrow t = 5$ or $t = 0$

Hence, zeroes are 5 & 0.

(ii) The maximum height is achieved at the vertex of the given parabola having $t = \frac{5}{2}$

$h\left(\frac{5}{2}\right) = 25\left(\frac{5}{2}\right) - 5\left(\frac{5}{2}\right)^2 = \frac{125}{4}$ m

(iii) (a) To reach 30m, $h = 30$ m

$30 = 25t - 5t^2$

$\Rightarrow 5t^2 - 25t + 30 = 0$

$\Rightarrow t^2 - 5t + 6 = 0$

$\Rightarrow (t-3)(t-2) = 0$

$\Rightarrow t = 3 \text{ or } t = 2$

Hence ball took 2 or 3 seconds.

OR

(b) $20 = 25t - t^2$

$\Rightarrow 5t^2 - 25t + 20 = 0$

$\Rightarrow t^2 - 5t + 4 = 0$

$\Rightarrow (t-4)(t-1) = 0$

$\Rightarrow t = 4 \text{ or } t = 1$

16. **ASSERTION REASON BASED QUESTIONS**

A statement of assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Assertion(A): If the graph of the polynomial $f(x) = ax^2 + bx + c, a > 0$ touches x-axis at its lowest point, then its zeroes are equal to $\left(-\frac{b}{2a}\right)$

Reason(R): If the graph of a quadratic polynomial touches x-axis, then its zeroes are equal.

Answer: (a)