Answer Key to PRACTICE QUESTIONS SET – 2 (2024-25)

MATHEMATICS

CLASS – X

Section A

Answer ALL the following Questions. Each question carries 2 marks.

1. **Two straight paths are represented by the equations $\mathbf{x - 3y = 2}$ and $\mathbf{-2x + 6y = 5}$. Check whether the paths cross each other or not.**

Answer:

Here, $\frac{a_1}{a_2} = -\frac{1}{2}$, $\frac{b_1}{b_2} = -\frac{1}{2}$, $\frac{c_1}{c_2} = \frac{2}{5}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, which is a condition of parallel lines.

Hence, two straight paths represented by the given equations never cross each other because they are parallel to each other.

2. **State whether $\mathbf{(x+1)(x-2)+x=0}$ has two distinct real roots. Justify your answer.**

Answer:

$(x+1)(x-2)+x=0 \Rightarrow x^2-2=0$

Discriminant $= b^2 - 4ac = (0)^2 - 4(1)(-2) = 8 > 0$

Hence, the equation $(x+1)(x-2)+x=0$ has two distinct real roots.

3. **Find the roots of the quadratic equation $\mathbf{\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0}$ using quadratic formula.**

Answer:

By quadratic formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-\sqrt{11}) \pm \sqrt{(-\sqrt{11})^2 - 4 \times \frac{1}{2} \times 1}}{2\left(\frac{i}{2}\right)}$

$= \sqrt{11} \pm \sqrt{9}$

$= \sqrt{11}+3, \sqrt{11}-3$

Therefore, $\sqrt{11}+3$ and $\sqrt{11}-3$ are the roots of the given equation.

4. **If $\mathbf{2x + y = 23}$ and $\mathbf{4x - y = 19}$, find the value of $\mathbf{(5y - 2x)}$ and $\mathbf{\left(\frac{y}{x} - 2\right)}$.**

Answer:

$2x + y = 23 - - - \text{(i)}$ and $4x - y = 19 - - - - \text{(ii)}$

Adding the equation (i) and (ii), 6x = 42 ⇒ x = 7.

Substituting x value in (i), we get, y = 9

Substituting the values of x and y in $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$, we get

$(5y - 2x) = 5 \times 9 - 2 \times 7 = 31$

And, $\frac{y}{x} - 2 = \frac{9}{7} - 2 = -\frac{5}{7}$

5. **Find the value(s) of k so that pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.**

Answer: For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\Rightarrow \frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$$

So, the given system of equations is consistent with unique solution for all values of k other than 6.

Section B

Answer ALL the following Questions. Each question carries 3 marks

6. **For which values of a and b will the pair of linear equations $x + 2y = 1$ and $(a - b)x + (a + b)y = a + b - 2$ has infinitely many solutions?**

Answer:

For infinitely many solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{a-b} = \frac{2}{a+b} = \frac{-1}{-(a+b-2)}$

$\frac{1}{a-b} = \frac{2}{a+b} \Rightarrow a = 3b$ ----------------(i)

$\frac{2}{a+b} = \frac{-1}{-(a+b-2)} \Rightarrow a + b = 4$ ------(ii)

Substituting equation (i) in equation (ii), we get $b = 1, a = 3$

As, $(a, b) = (3,1)$ satisfies all the parts, hence, $a = 3, b = 1$

7. Solve for x and y: $\frac{x}{a}+\frac{y}{b}=a+b,\ \frac{x}{a^2}+\frac{y}{b^2}=2,\ a, b\neq 0.$

Answer:

$\frac{x}{a}+\frac{y}{b}=a+b---$ (i) $\times\frac{1}{a}$ gives $\frac{x}{a^2}+\frac{y}{ab}=1+\frac{b}{a}----$(ii)

$\frac{x}{a^2}+\frac{y}{b^2}=2-----$ (iii)

Subtracting equation (ii) from equation (iii), we get

$y\left(\frac{1}{b^2}-\frac{1}{ab}\right)=2-1-\frac{b}{a}\Rightarrow\frac{y(a-b)}{ab^2}=\frac{a-b}{a}\Rightarrow y=b^2$

Substituting the value of y in equation (iii), we get

$\frac{x}{a^2}+\frac{b^2}{b^2}=2\Rightarrow x=a^2$

Hence, $x=a^2$ and $y=b^2$

8. A train, travelling at a uniform speed for 360 km, would have taken 48 min less to travel the same distance, if its speed were 5 km/hr more. Find the original speed of the train.

Answer:

Let the original speed of the train be x km/hr and the increased speed of the train be (x + 5) km/hr.

By the problem, $\frac{360}{x}-\frac{360}{x+5}=\frac{48}{60}$

$\Rightarrow\frac{[360(x+5)-360x]}{x(x+5)}=\frac{4}{5}$

$\Rightarrow\frac{1800}{x^2+5x}=\frac{4}{5}$

$\Rightarrow x^2+5x-2250=0$

$\Rightarrow x^2+50x-45x-2250=0$

$\Rightarrow(x+50)(x-45)=0$

$\Rightarrow x=-50$ (not possible) or $x=45$

Hence, the original speed of train is 45 km/hr,

9. Solve for x: $\frac{1}{2x-3}+\frac{1}{x-5}=1\frac{1}{9}\ , x\neq\frac{3}{2}\ , 5$

Answer:

$\frac{1}{2x-3}+\frac{1}{x-5}=\frac{10}{9}$

$\Rightarrow\frac{[(x-5)+(2x-3)]}{(2x-3)(x-5)}=\frac{10}{9}$

$\Rightarrow 9(3x - 8) = 10(2x - 3)(x - 5)$

$\Rightarrow 27x - 72 = 10(2x^2 - 13x + 15)$

$\Rightarrow 20x^2 - 157x + 222 = 0$

$\Rightarrow 20x^2 - 37x - 120x + 222 = 0$

$\Rightarrow (x - 6)(20x - 37) = 0$

$\Rightarrow x = 6$ or $x = \frac{37}{20}$

10. If the difference of the roots of the equation $x^2 - 7x + 2k = 0$ is 1 then find the value of k.

Answer:

Let α and β (α > β) are the roots of the equation $x^2 - 7x + 2k = 0$

$\Rightarrow \alpha + \beta = -\frac{-7}{1} = 7$ and $\alpha\beta = 2k$

By the problem, $\alpha - \beta = 1$

$\Rightarrow (\alpha - \beta)^2 = 1$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$

$\Rightarrow 7^2 - 4 \times 2k = 1$

$\Rightarrow k = 6$

Section C

Answer ALL the following Questions. Each question carries 5 marks.

11. A motorboat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21km upstream and return in 5 hours. Find the speed of boat in still water and the speed of the train.

Answer:

Let the speed of motorboat in still water and speed of the stream be x km/hr and y km/hr respectively.

By the problem,

Condition 1: Time taken upstream + time taken downstream = 7 hours

$\frac{30}{x-y} + \frac{28}{x+y} = 7 \Rightarrow 30a + 28b = 7 - - - (i)$

Condition 2: Time taken upstream + time taken downstream = 5hours

$\frac{21}{x-y} + \frac{21}{x+y} = 5 \Rightarrow 21a + 21b = 5 - - - (ii)$

(where, $\frac{1}{x-y} = a$ and $\frac{1}{x+y} = b$)--------(I)

$\text{eqn(i)} \times 3$ gives $90a + 84b = 21$ ----(iii)

$\text{eqn (ii)} \times 4$ gives $84a + 84b = 20$ --- (iv)

Subtracting we get $6a = 1 \Rightarrow a = \frac{1}{6}$ thus, $b = \frac{1}{14}$

From (I), $x - y = 6$ and $x + y = 14$

Solving, we get $x = 10$ and $y = 4$

Therefore, the speed of motorboat in still water and speed of the stream be 10 km/hr and 4 km/hr respectively.

12. If the sum of two roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero, then show that product of the two roots is $\left[-\frac{a^2+b^2}{2}\right]$.

Answer:

$\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

$\Rightarrow \frac{(x+b)+(x+a)}{(x+a)(x+b)} = \frac{1}{c}$

$\Rightarrow x^2 + (a + b - 2c)x + (ab - bc - ca) = 0$

Sum of roots = 0

$\Rightarrow -\frac{a+b-2c}{1} = 0 \Rightarrow c = \frac{a+b}{2}$

Product of roots = $\frac{ab-bc-ca}{1}$

$= ab - c(a + b)$

$= ab - \left(\frac{a+b}{2}\right)(a + b)$

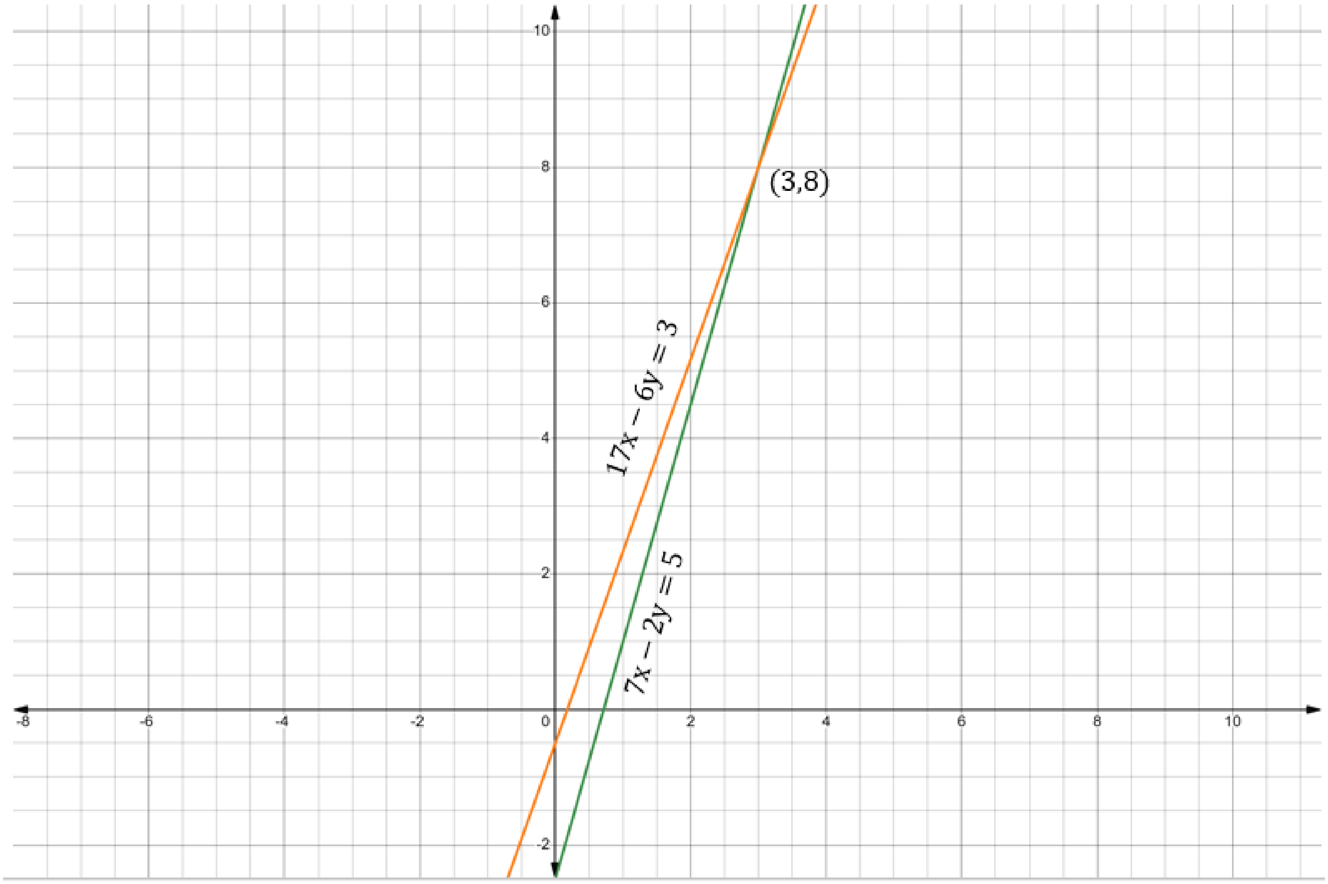
$= \frac{[2ab-(a+b)^2]}{2} = -\frac{(a+b)^2}{2}$

13. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3.

a) Represent the above problem in the form of the equations in two variables x, y (considering one's place digit x and ten's place digit y, where $y > x$).

b) Hence, solve the equations graphically to find the number.

Answer:



Let the digit at one's place be x and digit at ten's place be y.

By the problem,

$10y + x = 8(x + y) - 5 \Rightarrow 7x - 2y = 5 - - - (i)$

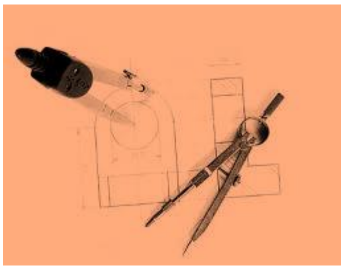
$10y + x = 16(y - x) + 3 \Rightarrow 17x - 6y = 3 - -(ii)$

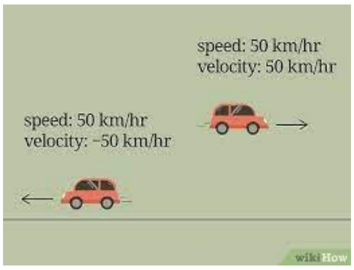
From the graph, $x = 3$ and then $y = 8$

Therefore, the number is 83.

Section – D: Case Study

14. Quadratic equations are used in many real-life situations such as calculating the areas of an enclosed space, the speed of an object, the profit and loss of a product, or curving a piece of equipment for designing.



a) If the roots of the quadratic equation are 2, −3, find the equation.

Answer: $(x - 2)(x + 3) = 0 \Rightarrow x^2 + x - 6 = 0$

b) If one root of a quadratic equation is $\frac{1+\sqrt{5}}{7}$, find the other root.

Answer: The other root is $\frac{1-\sqrt{5}}{7}$

c) If one root of the quadratic equation $x^2 + kx + 1 = 0$ is $\left(-\frac{1}{2}\right)$, find k.

Answer:

Since, $\left(-\frac{1}{2}\right)$ is the root of the equation so it satisfies the given equation.

$$\left(-\frac{1}{2}\right)^2 + k\left(-\frac{1}{2}\right) + 1 = 0 \Rightarrow k = \frac{5}{2}$$

OR

If the roots of quadratic equation $x^2 + mx + 12 = 0$ are in the ratio 1 : 3, find m.

Answer:

Let the roots be α and 3α

$$\Rightarrow \alpha + 3\alpha = -\frac{m}{1} \Rightarrow \alpha = -\frac{m}{4}$$

$$\Rightarrow \alpha \times 3\alpha = 12 \Rightarrow \alpha^2 = 4$$

$$\Rightarrow \left(-\frac{m}{4}\right)^2 = 4 \Rightarrow m = \pm 8$$

15. **ASSERTION REASON BASED QUESTIONS**

A statement of assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Assertion(A): If $ac \neq 0$, then atleast one of the two equations $ax^2 + bx + c = 0$ and $ax^2 + bx - c = 0$ has real and distinct roots.

Reason(R): A quadratic equation has real and distinct roots if the discriminant is positive.

Answer: (a)